



ELSEVIER

Journal of Public Economics 77 (2000) 81–95

JOURNAL OF
PUBLIC
ECONOMICS

www.elsevier.nl/locate/econbase

Endogenous authoritarian property rights

Jimmy Teng*

*Sun Yat-Sen Institute for Social Sciences and Philosophy, Academia Sinica, Nankang, Taipei,
Taiwan 11529, PR China*

Received 1 January 1996; accepted 1 May 1999

Abstract

Taxation and state enforcement of property rights share many common resources. Both are involved in determining the revenue of the state. Due to these interactions, taxation and property rights enforcement of a proprietary state are complementary. Any change in the administrative capacity of the state that increases the taxation rate also increases the level of property rights enforcement, and vice versa. The capacity and incentive of the proprietary state to tax are also essentially the capacity and incentive to enforce property rights. Extra enforcement powers of the proprietary state might therefore increase rather than decrease production. © 2000 Elsevier Science S.A. All rights reserved.

Keywords: Property rights; Taxation; Complementarity; Economies of scope; Proprietary state

JEL classification: D23; D72; H10

1. Introduction

The theory of the firm has dwelt in depth on the concepts of demand and strategic complementarity or substitutability, and the economies of scale and scope.¹ Literature on the state has applied some of these concepts. Dudley (1991) and Hirshleifer (1995) analyze some historical stylized facts concerning the state using the concept of economies of scale. Lane (1942) discusses the economies of

*Tel.: +886-2-2789-8138; fax: +886-2-2785-4160.

E-mail address: jim_teng@hotmail.com (J. Teng)

¹For example, see Chandler (1990).

scope between taxation and property rights enforcement. This paper attempts to study formally the interactions between two central state functions of taxation and property rights enforcement by using the concepts of complementarity and economies of scope.

There has been extensive analysis of the economic importance of a property rights regime (Demsetz, 1967; Bush, 1974; Skogh and Stuart, 1982; Yang and Wills, 1990; Skaperdas, 1992; Grossman and Kim, 1995; Hirshleifer, 1995). It is also recognized that both taxation and state property rights enforcement are important in deciding the revenue of the state and performance of the economy (Grossman, 1998). However, existing literature on property rights, taxation, and the proprietary state (Brennan and Buchanan, 1977; Grossman and Noh, 1994; Marcouiller and Young, 1995; McGuire and Olson, 1996; Grossman, 1997; Konrad and Skaperdas, 1998; McGuire, 1998) has not yet focused on analyzing the interactions between taxation and state enforcement of property rights.

The literature on the proprietary state has also not dealt explicitly with the economic implications of certain features of the cost function of taxation and state property rights enforcement. For instance, in Grossman and Noh (1994), Marcouiller and Young (1995), McGuire and Olson (1996), and Grossman (1997), taxation has a distorting effect on productive labor supply, creating deadweight losses. Taxation and public provision of order, however, incur no administration costs. This is unrealistic. Coercion, control, and monitoring of the population, like all other economic activities, require resources. Due to this oversight, there has not been any formal analysis of the economies of scope between taxation and state property rights enforcement. Other features of the cost function of taxation and property rights enforcement have also not been analyzed.

This paper fills in the above gaps by formally analyzing the interaction between taxation and property rights enforcement of a proprietary state in terms of revenue and cost. It brings in formally the cost function of taxation and state property rights enforcement. Complementarity characterizes the interactions between the two policies: a change in the administrative capacity of the state that increases the taxation rate also increases the level of property rights enforcement, and vice versa. The complementarity between taxation and state property rights enforcement has two components: revenue complementarity and economies of scope.

The state extracts revenue from the population through taxation. It uses part of this revenue to finance property rights enforcement. Property rights enforcement enhances the productivity of the economy and increases taxation revenue. This results in the revenue complementarity between taxation and property rights enforcement: increased property rights enforcement raises the marginal revenue of taxation, and vice versa. On the other hand, taxation and property rights enforcement share many common resources. For instance, both the information capacity and the coercion capacity of the state are essential for enforcing taxation and the property rights regime (Dudley, 1991). These are the economies of scope between taxation and property rights enforcement: increased taxation lowers the

marginal cost of property rights enforcement, and vice versa. With revenue complementarity and economies of scope between taxation and state property rights enforcement, the capacity and incentive to tax are also the capacity and incentive to enforce property rights.

This paper shows that the interactions between taxation and property rights enforcement due to revenue complementarity and economies of scope have important implications. We have a more complete understanding of issues on property rights, conflicts and the proprietary state when the interactions between taxation and state property enforcement are considered formally.

2. The model

This is a two-stage game with two types of players: the proprietary state and a large number of economic agents. The state maximizes its profits. The profits of the state are its gross tax revenue minus the costs of taxation and property rights enforcement. The state taxes the production of the agents and penalizes the predation of the agents. The agents maximize their net personal income.²

In stage one, the state sets its taxation rate and level of property rights enforcement.

In stage two, the agents allocate their labor endowment between production and preying on the production of others. The agents make their decisions simultaneously.

2.1. The agents

2.1.1. Assumptions

- (a) There are n identical atomistic agents. n is a large number.
- (b) The labor endowment of the agents is the only factor of production in the economy. Each agent is endowed with one unit of labor supply.³
- (c) The agents acquire income through production as well as through preying on the production of others.
- (d) The agents do not self enforce property rights.⁴
- (e) The production function is

$$P(l_i) = \alpha l_i - \beta (l_i)^2. \quad (1)$$

²For simplicity we assume the labor supply of the agents is fixed. Net income then measures utility.

³We focus on the allocation of labor endowment between production and predation. We ignore the issue of leisure.

⁴Endogenizing the amount of private enforcement of property rights, as in Grossman and Kim (1995), will not affect the central arguments of the paper.

We assume that $\alpha > 0$, $\beta > 0$ and $\alpha > 2\beta$. $P(l_i)$ refers to production of agent i . The subscript i denotes the identity of the agent. l_i refers to the labor input of agent i for production.

(f) The predation function is

$$S(1 - l_i) = S \times (1 - l_i).^5 \quad (2)$$

$S(1 - l_i)$ is the proportion of production of the other $(n - 1)$ agents that is preyed on by agent i . S is the efficiency parameter of predation; $S > 0$. $(1 - l_i)$ is the labor input of agent i for predation.⁶

(g) There is no external or self-sufficient sector. Secession from the economy is impossible.

(h) When producing, the agents are taxed by the state. The proportionate tax rate is t . When preying, the agents face property rights enforcement by the state. The state enforces property rights by restitution. It identifies production lost through predation, recovers it, and returns it to the owner. f is the restitution rate.⁷

(i) For the representative agent i , his net personal income acquired through production (net of loss from the predation of all other $(n - 1)$ agents and the taxation of the state) and preying on the production of all other $(n - 1)$ agents (net of the restitution effort of the state) is

$$y_i = \left(1 - t - (1 - f) \sum_{j \neq i}^{n-1} S(1 - l_j) \right) P(l_i) + (1 - f) S(1 - l_i) \sum_{j \neq i}^{n-1} P(l_j), \quad (3)$$

where y refers to the net personal income of the agent and the subscripts i and j denote the identity of the agents.

(j) In aggregate, predation does not create wealth. It merely transfers.⁸ The total income of the economy (Y) is therefore the sum of gross production of the agents:

$$Y = \sum_{i=1}^n P(l_i). \quad (4)$$

⁵Conflict functions (Hirshleifer, 1988, 1995; Skaperdas, 1992) analyze contests over resources among a small number of contestants. This paper analyzes the case of a large number of agents. There is a free rider problem in the enforcement of property rights. The participants find it worthwhile to prey on the production of others but not to enforce property rights.

⁶An alternative interpretation is that the model takes the amount of private enforcement of property rights as given and incorporates its effects in the parameter S .

⁷Restitution is used as a metaphor here. Our focus is on how the state changes the allocation of agents' effort between predation and production. The main conclusions of the paper remain unchanged if we use other measures of adequacy in property rights enforcement.

⁸Since the agents do not enforce property rights, predation does not cause damages or injuries. For simplicity, we also assume that agents do not resist the restitution effort of the state. Given this assumption, there are no damages or injuries during restitution as well. Allowing for destruction during predation or restitution will not change the main results.

2.1.2. The agents' decision

Given that all the other $n - 1$ identical agents have made their income-maximizing decisions on labor input into production and predation, denoted by \bar{l} and $(1 - \bar{l})$, agent i solves the Lagrangian function:

$$\max_{l_i} y_i = (1 - t - (1 - f)(n - 1)S(1 - \bar{l}))P(l_i) + (1 - f)S(1 - l_i)(n - 1)P(\bar{l}). \quad (5)$$

We concentrate our analysis on the interior equilibrium where the agents allocate their labor endowments between both predation and production, so that $0 < l < 1$.⁹

By the assumption of identical agents, we have $l_i = \bar{l}$ and $1 - l_i = (1 - \bar{l})$ at a symmetric equilibrium. We denote \bar{l} as l and $(1 - \bar{l})$ as $(1 - l)$ for notational convenience.

From solving Eq. (5), the first-order condition is

$$(1 - t - (1 - f)(n - 1)(1 - l)S)P' - (1 - f)S(n - 1)P = 0. \quad (6)$$

Solving Eq. (6) simultaneously for all the agents, we derive the following:

$$\frac{dY}{df} > 0, \quad (7)$$

$$\frac{dY}{dt} < 0, \quad (8)$$

$$\frac{\partial^2 Y}{\partial f^2} < 0, \quad (9)$$

$$\frac{\partial^2 Y}{\partial t^2} < 0, \quad (10)$$

$$\frac{\partial^2 Y}{\partial t \partial f} > 0. \quad (11)$$

For convenience, we adopt the following notational simplifications:

$$\frac{dY}{df} \equiv Y_f, \quad \frac{dY}{dt} \equiv Y_t, \quad \frac{\partial^2 Y}{\partial f^2} \equiv Y_{ff}, \quad \frac{\partial^2 Y}{\partial t^2} \equiv Y_{tt}, \quad \frac{\partial^2 Y}{\partial t \partial f} \equiv Y_{tf}.$$

From inequality (7), a higher restitution rate increases total production. A higher taxation rate decreases total production, as seen in inequality (8).

⁹The interior solution at a symmetric equilibrium is unique. For proof, check the slope of the reaction function of the labor decision of agent i with respect to the labor decision of other agents and its second derivative.

By changing the net returns from production and predation, taxation and restitution alter the agents' allocation of effort between predation and production.

From inequality (9), restitution has a diminishing positive effect on total production. From inequality (10), taxation has an increasing negative effect on total production. An increased restitution rate decreases the negative effect of taxation on total production and an increased taxation rate increases the positive effect of restitution on total production, as seen in inequality (11).¹⁰

The above results show that gross production depends on taxation and restitution rates:

$$Y = Y(t, f). \quad (12)$$

Proposition 1. *The establishment of a state increases production if the state enforces a restitution rate higher than the tax rate, and conversely.*

Proof. From Eq. (6), production with the proprietary state takes place at

$$\frac{(1-t)}{(1-f)(n-1)S} = (1-l) + \frac{P}{P'}.$$

Under anarchy, $t = 0$ and $f = 0$. Production under anarchy therefore takes place at

$$\frac{1}{(n-1)S} = (1-l) + \frac{P}{P'}.$$

Differentiating the right-hand side of both equations above with respect to l , we have

$$\frac{\partial((1-l) + (P/P'))}{\partial l} = -\frac{P''P}{(P')^2} > 0,$$

where $P'' = -2\beta < 0$ by assumption (e). The right-hand side of the two equations above are monotonically positively related to l and P . Production under the proprietary state is greater than production under anarchy if $((1-t)/(1-f)) > 1$. Proposition 1 therefore holds. \square

Taxation and restitution alter the net returns of production and predation. The relative net returns of production and predation determine the agents' allocation of labor endowment between production and predation.¹¹ When the restitution rate exceeds the taxation rate, relative net returns from production under the state exceed those under anarchy. As a result, the agents devote more effort to production and less effort to predation.

¹⁰These results are due to the concavity of the production function. See Appendix A for details.

¹¹Refer to inequalities (7) and (8).

2.2. The state

2.2.1. Assumptions

- (a) The proprietary state is the only law making and enforcement agency in the defined territory.
- (b) The state is autocratic. It does not recruit its members from the population at large.¹²
- (c) The state does not participate in production. The state maximizes its profits through taxation and enforcement of property rights (restitution).
- (d) The costs of taxation and restitution are denoted by

$$C(t, f) = at + bf - ktf + G(t) + H(f), \quad a > 0, b > 0, k > 0. \quad (13)$$

A larger a (b) means that the state is less efficient in taxation (restitution). The extent of economies of scope between taxation and restitution ($\partial C / \partial t \partial f$) is measured by k .¹³

There are policy specific costs: $a > k$ and $b > k$.

$G(t)$ and $H(f)$ are functions with well-defined first and second derivatives, where $\partial G / \partial t \geq 0$, $\partial H / \partial f \geq 0$, $\partial^2 G / \partial t^2 \geq 0$, and $\partial^2 H / \partial f^2 \geq 0$. The value of $\partial^2 G / \partial t^2$ and $\partial^2 H / \partial f^2$ measures the speed at which marginal administrative cost rises with increasing taxation and restitution rates. The greater the second derivative $\partial^2 G / \partial t^2$ ($\partial^2 H / \partial f^2$), the faster the marginal administrative cost rises with increasing taxation (restitution) rate.¹⁴

For convenience, we adopt the following notational simplifications:

$$\frac{\partial C}{\partial t \partial f} \equiv k \equiv C_{tf}, \quad \frac{\partial G}{\partial t} \equiv G_t, \quad \frac{\partial H}{\partial f} \equiv H_f, \quad \frac{\partial^2 G}{\partial t^2} \equiv G_{tt}, \quad \frac{\partial^2 H}{\partial f^2} \equiv H_{ff}.$$

2.2.2. The state's decision

To maximize its profits, the state solves

$$\max_{t, f} tY(t, f) - at - bf + ktf - G(t) - H(f) \quad (14)$$

subject to the income maximization of the agents.

Differentiating Eq. (14) with respect to t yields

$$tY_t + Y - a + kf - G_t = 0 \quad (15)$$

and differentiating Eq. (14) with respect to f results in

¹²This simplification exempts us from endogenizing the size of the state and the economy.

¹³For a detailed analysis of the costs of property rights enforcement, see Stigler (1970).

¹⁴Marginal administrative cost could rise for the following reasons: (1) diminishing returns, (2) diseconomies of scale, (3) increasing input prices, (4) capacity constraints, and (5) constitutional or political constraints.

$$tY_f - b + kt - H_f = 0. \quad (16)$$

We denote the optimal taxation and restitution rates of the proprietary state by t^* and f^* . The corresponding level of total production is denoted by Y^* . We analyze the interior solution where $0 < t^* < 1$ and $0 < f^* < 1$.¹⁵

Solving Eqs. (15) and (16) with respect to changes in efficiency in taxation (a) and restitution (b), we have:

Proposition 2. *The optimal taxation and restitution rate for the proprietary state are complementary. An increase in the taxation efficiency of the state raises the restitution rate. An increase in the restitution efficiency of the state raises the taxation rate.*¹⁶

This complementarity is best understood by examining inequality (A.11):

$$\frac{df^*}{da} = \frac{dt^*}{db} = -\frac{1}{|J|}(tY_{tf} + Y_f + k) = -\frac{1}{|J|}(MR_{tf} - C_{tf}) < 0,$$

where $|J|$ is the Jacobian determinant. MR is marginal revenue.

$-C_{tf} = k > 0$: This is the economies of scope between taxation and restitution. Any increase in taxation rate lowers the marginal cost of restitution and vice versa.

$MR_{tf} = \partial MR_f / \partial t = \partial MR_t / \partial f = (Y_f + tY_{tf}) > 0$: An increased tax rate raises the marginal revenue from restitution and vice versa. This is the revenue complementarity between taxation and restitution.¹⁷

The revenue complementarity can be broken down into two parts:

- (a) tY_{tf} arises because of interaction between taxation and restitution on production.¹⁸
- (b) Y_f arises since more restitution raises the tax base.¹⁹

Due to the revenue complementarity and economies of scope between taxation and restitution, an increase in taxation efficiency raises the restitution rate and vice versa.

¹⁵For the uninteresting case of costless administration, the model has an implausible implication where both taxation rate and restitution rates are set to one. This is due to the assumption that there is no secession from the economy.

¹⁶Mathematically, if the off-diagonal elements of the two by two Jacobian determinant are positive (negative), the two policy instruments are complements (substitutes). Refer to Appendix A for details.

¹⁷See Eq. (13).

¹⁸See inequality (11).

¹⁹See inequality (7).

Given the complementarity between taxation and restitution, we have Propositions 3 and 4.

Proposition 3. *If increasing the restitution rate causes a substantial rise in the marginal administrative costs, better taxation efficiency then reduces total production. If marginal administrative costs do not rise substantially when the restitution rate is raised, better taxation efficiency then raises total production.*²⁰

Better efficiency in taxation raises both the taxation rate and restitution rate. Higher taxation reduces production. On the other hand, a higher restitution rate increases production. The net effect on production depends on the relative size of adjustment in the rates of taxation and restitution.

If raising the restitution rate increases the administrative costs substantially, the increase in the restitution rate is small. The negative effect on production due to increased taxation overwhelms the positive effects of increased restitution. Thus, better efficiency in taxation raises the tax rate and lowers production, and makes agents worse off.²¹

Increased ability by the government to extract resources from the producers may increase production. The state can extract resources from both the producers (taxation) and the predators (restitution). So long as the state accompanies its increased taxation with a higher restitution rate, production need not be lowered.

Proposition 4. *If the economies of scope between taxation and restitution are extensive, better efficiency in restitution then reduces production.*

*If the economies of scope between taxation and restitution are scanty, better efficiency in restitution then increases production.*²²

Better efficiency in restitution raises both the taxation and restitution rates. The effect on production depends on the degree of economies of scope between taxation and restitution. When there are extensive economies of scope, the increase in the tax rate is large. The negative effect of a higher tax rate overwhelms the positive effect of a higher restitution rate. Better efficiency in restitution therefore decreases production. With a higher tax rate and a lower production, agents are worse off.²³

²⁰Refer to Appendix A for proof.

²¹When raising the restitution rate does not increase the administrative costs substantially, we are not sure whether the net of tax income of the agents increases or decreases.

²²Refer to Appendix A for proof.

²³When the economies of scope are scanty, we are not sure whether the net of tax income of the agents increases or decreases.

3. Conclusion

Viewed in isolation, the taxation capacity of the state is generally considered to affect the economy negatively (Brennan and Buchanan, 1977; Marcouiller and Young, 1995). Taxation causes distortions in the economy. Viewed in isolation, the property rights enforcement capacity of the state is almost unanimously considered to affect the economy positively (Grossman and Kim, 1995; McGuire and Olson, 1996). Property rights enforcement curbs predatory activities and encourages production.

If an increase in state capacity to tax causes distortions and reduces the efficiency of the economy, then how do we explain the concomitant rise of the modern states and the modern economies in Europe and elsewhere? The modern states have much greater taxation capacity than their medieval predecessors (Webber and Wildavsky, 1986).

If an increase in state capacity to enforce property rights improves economic performance, then how could one explain the disappointing economic performance of the post-WWII welfare states? These states have great capacity for control and enforcement due to the build up of state capacity during WWII (Tilly, 1992). ‘English disease’ is the term that describes the sluggishness of these states (Olson, 1982).

This paper shows that taxation and state property rights enforcement should not be studied in isolation, given their revenue complementarity and economies of scope. The capacity and incentive of the proprietary state to tax are also essentially the capacity and incentive to enforce property rights. An increase in state capacity that leads to a higher taxation rate also results in a higher level of property rights enforcement. As a result, we do not know a priori the net effect on total production level.

To determine the effect of an increase in state capacity to tax and enforce property rights on total production, we need more information on the cost functions of taxation and property rights enforcement. Specifically, we need to know the degree of economies of scope between taxation and property rights enforcement and the marginal administrative costs facing the state in raising the level of property rights enforcement.

Better efficiency in taxation does not necessarily reduce production. If the marginal administrative costs do not rise sharply when the state raises the level of property rights enforcement, the accompanying increase in the level of property rights enforcement is large. Better efficiency in taxation then increases production. States with more enforcement power therefore do not necessarily have a lower production.

The establishment of large standing armies in Europe greatly increased the taxation capacity of the European states and was associated with great economic progress (Tilly, 1992). Concomitant with the rise of the large standing armies and powerful bureaucracies was the rediscovery and development of the Roman law,

which culminated in the Code Napoleon. These improvements in jurisprudence greatly aided the refinement of the property rights regime. The positive effects of a better property rights regime overwhelmed the negative effects of a higher taxation (North and Thomas, 1973).

Better efficiency in property rights enforcement does not necessarily increase gross output. If property rights enforcement and taxation have very extensive economies of scope, then the accompanying increase in the tax rate is large. Better efficiency in property rights enforcement then reduces production.

During World War II, the Western states built up their administrative capacity. They have therefore become more efficient in enforcing property rights. These states, however, also raised their taxation, especially for financing welfare programs. The resulting distortions due to higher taxation adversely affected their economic performance (Lindert, 1994).

To conclude, it is important that we understand how different policies of the state interact. One aspect of the interaction between different state policies is the complementary or substitute relationship examined in this paper. We need this understanding for evaluating the economic effects of events that affect state capacity. Examples include the dismantling of the state machinery in the former Soviet Union and the reform of China. This understanding is also important for evaluating proposals to constrain or augment state capacity in certain policy areas, such as the suggestion by Brennan and Buchanan (1977) to limit the state's taxation capacity.

Acknowledgements

I would like to thank D.M. Nowlan, R.M. Bird, J.M. Mintz, R.H. Gordon, L. Dudley, P.Y. Lai, J. Munro, M. Smart, M. Wooders, Y. Kotowitz and two anonymous referees for comments. I would also like to thank the Lee Foundation of Singapore for a graduate scholarship at the University of Toronto.

Appendix A

The agents' decision

Agent i solves

$$\max_{l_i} y_i = (1 - t - (1 - f)(n - 1)S(1 - \bar{l}))P(l_i) + (1 - f)S(1 - l_i)(n - 1)P(\bar{l}). \quad (\text{A.1})$$

The first-order condition is

$$(1 - t - (1 - f)(n - 1)S(1 - \bar{l}))P' - (1 - f)S(n - 1)P(\bar{l}) = 0. \quad (\text{A.2})$$

The second-order condition is

$$P''(1 - t - (1 - f)(n - 1)(1 - \bar{l})S) < 0.$$

Solving the problem of all the agents simultaneously, we have

$$\begin{aligned} \frac{dl}{df} (\equiv l_f) &= - \frac{\partial^2 Y_i / \partial l_i \partial f}{\partial^2 Y_i / \partial l_i^2} = \frac{-(n - 1)S((1 - l)P' + P)}{P''(1 - t - (1 - f)(n - 1)(1 - l)S)} > 0, \\ Y_f &= nP'l_f > 0, \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} \frac{dl}{dt} (\equiv l_t) &= - \frac{\partial^2 Y_i / \partial l_i \partial t}{\partial^2 Y_i / \partial l_i^2} = \frac{P'}{P''(1 - t - (1 - f)(n - 1)(1 - l)S)} < 0, \\ Y_t &= nP'l_t < 0, \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} Y_{ff} &= n \left(\frac{1}{P''(1 - t - (1 - f)(n - 1)(1 - l)S)} \right)^2 (P'')((n - 1)S)^2((1 - l)P' \\ &\quad + P)\{2P'(1 - l) + P\} + P'\{(1 - l) + (1 - f)l_f\} < 0, \end{aligned} \quad (\text{A.5})$$

$$\begin{aligned} Y_{tt} &= n \left(\frac{1}{P''(1 - t - (1 - f)(n - 1)(1 - l)S)} \right)^2 (P'')\{3(P')^2 \\ &\quad - (P')^2(1 - f)(n - 1)Sl_t\} < 0, \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} Y_{tf} &= n \left(\frac{1}{P''(1 - t - (1 - f)(n - 1)(1 - l)S)} \right)^2 (P'')(n - 1)S\{-2P'(P) \\ &\quad - 3(P')^2(1 - l) - (P')^2(1 - f)l_f\} > 0. \end{aligned} \quad (\text{A.7})$$

The state's decision

The state solves

$$\max_{t,f} tY(t,f) - at - bf + ktf - G(t) - H(f) \quad (\text{A.8})$$

subject to the income maximization of the agents.

The second-order condition requires that the Jacobian determinant be positive:

$$\begin{aligned} |J| &= \begin{vmatrix} (tY_{tt} + 2Y_t - G_{tt}) & (tY_{tf} + Y_f + k) \\ (tY_{tf} + Y_f + k) & (tY_{ff} - H_{ff}) \end{vmatrix} \\ &= t^2(Y_{tt}Y_{ff} - (Y_{tf})^2) + 2t(Y_tY_{ff} - Y_fY_{tf}) - (Y_f)^2 \\ &\quad - 2ktY_{tf} - 2kY_f - k^2 - tY_{ff}G_{tt} - tY_{tt}H_{ff} - 2Y_tH_{ff} + G_{tt}H_{ff}. \end{aligned}$$

The first six terms of the above expression are negative, where

$$(Y_{tt}Y_{ff} - (Y_{tf})^2) = \left(n \left(\frac{1}{P''(1-t-(1-f)(n-1)(1-l)S)} \right)^2 (P'')(n-1)S \right)^2$$

$$(-P'P^2) < 0$$

and

$$(Y_{tt}Y_{ff} - Y_fY_{tf}) = (n^2) \left(\frac{1}{P''(1-t-(1-f)(n-1)(1-l)S)} \right)^3$$

$$\{(P')^2(n-1)SP l_f(P'')^2(1-t-(1-f)(n-1)(1-l)S)\} < 0.$$

To have a positive Jacobian determinant we need either a large positive G_{tt} or H_{ff} or both.

Using the implicit function rule and the Cramer's rule, we have

$$\frac{dt^*}{da} = \frac{1}{|J|}(tY_{ff} - H_{ff}) < 0, \quad (\text{A.9})$$

$$\frac{df^*}{db} = \frac{1}{|J|}(tY_{tt} + 2Y_t - G_{tt}) < 0, \quad (\text{A.10})$$

$$\frac{df^*}{da} = \frac{dt^*}{db} = -\frac{1}{|J|}(tY_{tf} + Y_f + k) < 0. \quad (\text{A.11})$$

Proof of Proposition 3.

$$\frac{dY^*}{da} = Y_t \frac{dt^*}{da} + Y_f \frac{df^*}{da}$$

$$= \frac{1}{|J|}(Y_t(tY_{ff} - H_{ff}) - Y_f(tY_{tf} + Y_f) + k)$$

$$= \frac{1}{|J|}(t(Y_tY_{ff} - Y_fY_{tf}) - Y_f^2 - Y_fk - Y_tH_{ff}),$$

where $(Y_tY_{ff} - Y_fY_{tf}) < 0$.

In $(t(Y_tY_{ff} - Y_fY_{tf}) - Y_f^2 - Y_fk - Y_tH_{ff})$, the first three terms are negative. The fourth term is non-negative.

dY^*/da is therefore negative for a small $H_{ff} < (t(Y_tY_{ff} - Y_fY_{tf}) - Y_f^2 - Y_fk)(1/Y_t)$ and positive for a large $H_{ff} > (t(Y_tY_{ff} - Y_fY_{tf}) - Y_f^2 - Y_fk)(1/Y_t)$. \square

Proof of Proposition 4.

$$\begin{aligned}
\frac{dY^*}{db} &= Y_t \frac{dt^*}{db} + Y_f \frac{df^*}{db} \\
&= \frac{1}{|J|} (-Y_t(tY_{tf} + Y_f + k) + Y_f(tY_{tt} + 2Y_t - G_{tt})) \\
&= \frac{1}{|J|} (t(Y_f Y_{tt} - Y_t Y_{tf}) + Y_t Y_f - Y_f G_{tt} - Y_t k),
\end{aligned}$$

where

$$\begin{aligned}
(Y_f Y_{tt} - Y_t Y_{tf}) &= (n^2) \left(\frac{1}{P''(1-t-(1-f)(n-1)(1-l)S)} \right)^3 \\
&((- (n-1)SP)(P')^3 P'') < 0.
\end{aligned}$$

In $(t(Y_f Y_{tt} - Y_t Y_{tf}) + Y_t Y_f - Y_f G_{tt} - Y_t k)$, the first three terms are negative. The fourth term is non-negative.

dY^*/db is therefore positive for large $k > (t(Y_f Y_{tt} - Y_t Y_{tf}) + Y_t Y_f - Y_f G_{tt})(1/Y_t)$ and negative for small $k < (t(Y_f Y_{tt} - Y_t Y_{tf}) + Y_t Y_f - Y_f G_{tt})(1/Y_t)$. \square

References

- Brennan, G., Buchanan, J.M., 1977. Towards a tax constitution for Leviathan. *Journal of Public Economics* 8, 255–273.
- Bush, W.C., 1974. Some implications of anarchy for the distribution of property. *Journal of Economic Theory* 8, 401–412.
- Chandler, Jr. A.D., 1990. *Scale and Scope: The Dynamics of Industrial Capitalism*, Belknap, Cambridge, MA.
- Demsetz, H., 1967. Toward a theory of property rights. *American Economic Review Proceedings*, 347–359.
- Dudley, L., 1991. *The Word and the Sword: How Techniques of Information and Violence Have Shaped Our World*, Basil Blackwell, Cambridge, MA.
- Grossman, H.I., Noh, S.J., 1994. Proprietary public finance and economic welfare. *Journal of Public Economics* 53, 187–204.
- Grossman, H.I., Kim, M., 1995. Swords or plowshares? A theory of the security of claims to property. *Journal of Political Economy* 103 (6), 1275–1288.
- Grossman, H.I., 1997. Make us a king: anarchy, predation, and the state. Working paper, Department of Economics, Brown University.
- Grossman, H.I., 1998. Producers and predators. Working paper, Department of Economics, Brown University.
- Hirshleifer, J., 1988. The analytics of continuing conflict. *Synthese* 76, 201–233.
- Hirshleifer, J., 1995. Anarchy and its breakdown. *Journal of Political Economy* 103 (1), 26–52.
- Konrad, K.A., Skaperdas, S., 1998. The market for protection and the origin of the state. Working paper, Department of Economics, University of Irvine.
- Lane, F.C., 1942. Economic consequences of organized violence. *The Journal of Economic History* December, 401–417.

- Lindert, P.H., 1994. The rise of social spending, 1880–1930. *Explorations in Economic History* 31, 1–37.
- Marcouiller, D., Young, L., 1995. The black hole of graft: the predatory state and the informal economy. *American Economic Review* 85 (3), 630–646.
- McGuire, M.C., 1998. Redistribution, political power, and public goods. Irvine Economics Paper No. 98-99-04.
- McGuire, M.C., Olson, M., 1996. The economics of autocracy and majority rule: the invisible hand and use of force. *Journal of Economic Literature* XXXIV, 72–96.
- North, D., Thomas, R., 1973. *The Rise of the Western World: A New Economic History*, Cambridge University Press, Cambridge.
- Olson, M., 1982. *The Rise and Decline of Nations*, Yale University Press, New Haven, CT.
- Skaperdas, S., 1992. Cooperation, conflict, and power in the absence of property rights. *American Economic Review* September, 720–739.
- Skogh, G., Stuart, C., 1982. A contractarian theory of property rights and crime. *Scandinavian Journal of Economics* 84 (1), 27–40.
- Stigler, G.J., 1970. The optimum enforcement of laws. *Journal of Political Economy*, 526–536.
- Tilly, C., 1992. *Coercion, Capital, and European States, AD 990–1992*, Blackwell, Cambridge, MA.
- Yang, X., Wills, I., 1990. A model formalizing the theory of property rights. *Journal of Comparative Economics* 14, 177–198.
- Webber, C., Wildavsky, A., 1986. *A History of Taxation and Expenditure in the Western World*, Simon and Schuster, New York.